The A Level Content Advisory Board

REPORT OF THE ALCAB PANEL ON MATHEMATICS AND FURTHER MATHEMATICS

July 2014

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Executive summary

The aim of the recommended revisions in mathematics content is to provide modern A levels that contain necessary material and that are also interesting to learn and teach. Although we have recommended significant changes, we were not seeking to make the qualifications harder. The ALCAB mathematics panel recommends that the content of the single mathematics A level be fully prescribed and has gone into considerable detail with the recommended content. The content itself is not radically different from the existing A level but making it a requirement would ensure consistency across awarding organisations. On a practical level it is key that co-teaching of pure mathematics between the single A level and AS level further mathematics should be achievable and the detailed content reflects this aim.

The AS levels in mathematics and further mathematics are welcomed by university departments and have considerable value in their own right. They should be supported and retained.

Further mathematics qualifications will contain a minimum amount of prescribed pure mathematics: 30% at AS level and 50% at A level. Flexibility has been left to allow students to specialise, awarding organisations to innovate and new strands to be introduced if needs be.

Background

Over the last decade the number of students taking A level mathematics has risen very substantially, from 50,612 in 2003 to 88,060 in 2013, an increase of 74%. There has been an even greater proportional increase in the uptake of further mathematics over the same period, from 5,315 to 13,821, an increase of 160%. The numbers taking these A levels are the highest on record.

However, it is still the case that admissions tutors for a number of important user subjects, such as chemistry, economics and computer science, are not able to fill their places with students who have taken A level mathematics, even in highly

rated universities. So welcome though the increases are, there is still a need for more students to take A level mathematics.

The same is true for further mathematics, where the key user subjects in higher education are mathematics, engineering and physics. Thus the proportion of new physics undergraduates who have taken further mathematics has risen from 11% to 30%, but still has a long way to go. Consequently we have been very conscious of the need to sustain the increases in numbers taking both A levels and the momentum that has built up with year-on-year increases.

There is widespread concern that the changes planned for A levels will reduce the number of those taking mathematics, and especially of those taking further mathematics. These concerns are highlighted in the Review of Specification Content of July 2013 that states “New qualifications will have to be carefully designed so that uptake of subject continues at existing levels”. There are also concerns over the scale of change required with the moves to linear qualifications. Since these topics range wider than the content of the qualifications we will convey them in a separate letter.

At the outset we identified a number of perceived problems with the current provision which university departments of mathematics would wish to see overcome or ameliorated:

- The mathematical thinking of the most able students is not developed.
- The distinction between A and A* grades seems based on the avoidance of careless slips rather than genuine mathematical ability, making it hard for admissions tutors to pick out the students with the greatest potential. The A* should be awarded for demonstrating understanding and flair, not the ability to do routine calculations accurately, and the assessments should be developed accordingly.
- It is not clear what applied mathematics students have learnt.
- Current statistics provision tends to focus on routine calculations at the expense of interpretation and understanding.

The Review of Specification Content of July 2013 also highlights the role that mathematics takes in the support of other disciplines, the requirement for problem solving skills, the core content for stand-alone AS levels, and the content of further mathematics. Our report aims to address these points.

However, only about 10% of those taking A level mathematics go on to become mathematics undergraduates; for most of the other 90% it will be a service subject. That raises quite different problems:

- The numbers taking A level mathematics are insufficient for the needs of higher education.
- Some mathematics students do not develop transferable skills.

Meeting the concerns of both groups of end-users is difficult to achieve within a single A level; some would argue that their needs are mutually incompatible. However, we believe that the existence of further mathematics qualifications makes it possible to design a provision that addresses both. The support and nurturing of further mathematics must therefore be taken very seriously.

Report of the ALCAB panel on mathematics and further mathematics

Introduction

The ALCAB mathematics panel has met on six occasions (29 January, 14 February, 14 March, 24 April, 16 May, 30 May) since its formation early in 2014 and it formed two sub-panels - on mechanics and statistics - that met separately and reported back. At the initial meeting, orientation was provided by the DfE, Ofqual and the awarding organisations. We subsequently received advice from a representative of the Royal Statistical Society and members have reported upon their consultations with university departments and schools. Panel members met with teachers and educators at the British Congress of Mathematics Education (BCME) on 15 April and the Heads of Departments of Mathematical Sciences at their annual conference in Birmingham on 24 April. The Heads of Departments of Mathematical Sciences completed a questionnaire survey on the main issues being considered by the panel.

Individual panel members consulted the following bodies on behalf of the panel: Advisory Committee on Mathematics Education (ACME); Institute of Physics; Institute of Mathematics and its Applications; London Mathematical Society; Royal Society; Royal Academy of Engineering/Engineering Professors Council; SCORE and the Royal Statistical Society. The Operational Research Society attended a panel meeting and gave feedback upon planned reform. The various submissions and feedback have been discussed at length. There has therefore been extensive discussion about the content required.

Our view is that content is one part of a set of connected issues that involve assessment, delivery, structure and ultimately governance of A level examinations. A coordinated, coherent and long-term strategy should be developed for A level mathematics that is consistent with the aim of educating and engaging students with high-level material. The aim of the recommended revisions in content is not to make A levels harder, but to provide modern A levels that contain necessary material and that are also interesting to learn and teach; the quantity of content has not been increased. In doing so we have followed expert advice such as that from the Advisory Committee on Mathematics Education (ACME): "ACME believes that relatively little reform of the A level content is required, but that there should be improved quality of assessment with changes made incrementally over time. This is hard to achieve with the current regulatory structure and when there are competing awarding organisations." (A level reform position statement, 16 May 2013.)

Core content

The panel recommends that **100 per cent of the content for the single A level in mathematics should be fully prescribed centrally**. This recommendation follows consultation, in particular, with university departments, which wish to know what students should have covered. A typical submission, from a group of university admissions tutors is that "an identical syllabus between exam boards is required so we have a level of certainty that someone taking A level mathematics has knowledge of a certain set of topics to a given standard." The current range of options means that universities cannot be certain what their incoming A level students have studied. Teachers and educators at BCME did not oppose this proposal, recognising that in many institutions the current flexibility and choice does not extend to students. However, support for a fully prescribed A level is not universal and concern has been expressed that this might make the subject less attractive to some students, for example, those whose other A level subjects are psychology or business studies and who might want mathematics options available which were linked to these subjects.

Students studying for A level mathematics should learn a variety of mathematical concepts, methods and techniques. Some of what is studied can be unequivocally described as pure mathematics and other parts as applied, but there will be overlap and interplay between the two. To gain a good and useful understanding of the mathematical content, students will need to work at a variety of problems. In doing so they should learn to present mathematics clearly and logically, using mathematical terminology correctly and drawing graphs and other diagrams whenever this is helpful.

As mathematics often acts to support other disciplines and to address concerns raised in many disciplines about the need for students to have better problem-solving skills, **we are suggesting a change in emphasis within the single A level in mathematics towards problem solving, interpretation and testing understanding. This should drive assessment with less structured questions that test understanding and help to develop strategies for solving problems either in a purely mathematical or in an applications context.**

This change in emphasis is motivated by several submissions such as that from the Engineering Professors' Council: "The linkage between applications and options should also be reconsidered. As the considerable majority of those who take mathematics A level and progress on to higher education do not go on to study

mathematics at university we believe the interests of learners as a whole would be better served if examples of the wide applicability of the mathematics they are learning were encountered throughout their study of the subject. Such examples help motivate students and contribute to understanding".

Further Mathematics

For further mathematics the panel considers that a core of 50% should be prescribed. This will then leave flexibility for students wishing to progress to, say, engineering or physics to specialise in mechanics and for those wishing to study, say, economics to specialise in statistics. It will also allow awarding organisations the freedom to develop distinctive strands, react to emerging technology and innovate.

AS level mathematics and further mathematics

The AS level qualifications in mathematics and further mathematics are both worthwhile and valuable, particularly for many students aiming for quantitative degree programmes.

We have discussed the arrangement of the material for the AS in single mathematics to try to ensure that A level students can be taught the AS material in their first year, ensuring that separate classes are not needed. We have again suggested that this content be fully prescribed and indicated which topics would be most suitable.

For the AS in further mathematics, which is valued by many university departments, for example in engineering, physics and economics, we have suggested that **a minimum of 30% of the content must be drawn from the prescribed material of the A level in further mathematics and must include matrices and complex numbers.**

Decision mathematics

Modules entitled 'Decision mathematics' have been available within A level mathematics for 15 years. However, they are not universally valued by end users; in the words of a Head of Computer Science at a Russell Group university: ‘‘We discuss what we will do when schools are actually teaching all our students programming (about 40 per cent come in knowing how to program). We have never

discussed whether we will ask them to take Decision Maths. We see Decision Maths as soft modules that take the place of maths we wish them to know." Feedback received from economics, computing and mathematics departments was along similar lines. For instance, a survey of the Heads of Departments of Mathematical Sciences (HoDoMS) had the usefulness of Decision Mathematics as a question: the result was that 14 ½ found it not useful versus 1 ½ who found it useful. As a result **we are recommending that this content be completely removed from A level mathematics**. There was a submission to the panel from the Operational Research Society who viewed it as valuable, if in need of overhaul. The view that it is a soft option is disputed by the awarding organisations and Ofqual on statistical grounds

There is potentially a place in further mathematics for a serious strand of mathematics based on discrete mathematics and this could be considered as an additional strand alongside mechanics and statistics. However, this will require scrutiny to ensure that it will be perceived as a valuable part of further mathematics.

Statistics

In our view, assessment items in statistics should not have, as their primary focus, routine calculations of summary statistical measures or the drawing by hand of statistical diagrams. **The use of real, large data sets should permeate the teaching, learning and assessment of statistics in AS and A level mathematics and further mathematics.**

The existing content has therefore been modified in the light of recommendations from the Royal Statistical Society, professional statisticians, and from the university departments that we surveyed. **It will place more emphasis upon understanding, interpretation of data and making inferences from data**. The use of large pre-released data sets is suggested. This may involve a change in classroom practice for some teachers and resultant training will be needed.

Mechanics

Historically developments in applied mathematics have at times followed developments in pure mathematics, and at other times driven them. A similar two-way process occurs in learning mathematics. Again, the panel feels that **an emphasis upon understanding, interpretation and problem solving should**

pervade the teaching and assessment of mechanics; there are many natural linkages with the pure mathematics content and these linkages should be exploited. The mechanics content of the A levels in mathematics and further mathematics are particularly valued by the natural sciences and engineering and naturally provide situations in which to develop modelling skills.

The general philosophy for the mechanics content is that **during their study students will build up understanding of the mathematical modelling process, including recognising the assumptions made and how these might be modified to improve the model used.** This understanding will grow organically from their encounter with particular topics, and will increase in sophistication. It should include an awareness that some laws are obtained by experiment, while others may be derived from more basic laws. It should also include an ability to handle approximations and to reflect critically on the results of the modelling process and suggest possible improvements.

Mechanics within mathematics is naturally related to pieces of A level physics and **a concerted effort should be made to ensure that examining organisations use common terminology between mathematics and physics where possible**. This was emphasised in oral submissions from SCORE and the Institute of Physics. A position paper from the Engineering Professors' Council, states in the context of synchronisation that "this is particularly important in the context of mechanics which features in the specifications of Mathematics, Further Mathematics and Physics A Levels. If these subjects are properly coordinated there is considerable scope thereby to support and strengthen advantageously the mathematical sophistication of Physics A level. This opportunity should not be missed". These representations all support the panel's conclusion that mechanics is important and should be thought of in a wider context.

Technology in assessment

There are conflicting arguments about the appropriate use of technology in assessment. On the one hand, it is undesirable to have an out-dated examination requiring the use of tables that enforce old-fashioned content and teaching. On the other hand, it is potentially unfair to allow sophisticated expensive graphing calculators that not all students can afford to buy. **We have recommended the use of technology in both teaching and assessment and for this to take place successfully it must be adequately resourced.**

One possibility is the creation of, say, an annually updated list of allowed calculators (as is done for the International Baccalaureate) although that might be hard to enforce.

A different issue is whether calculators should be disallowed from some examination papers. Experience has shown that this can distort the syllabus and the questions that can be set. A better way forward would be for some examination questions to start with an instruction like “You may not use a calculator on this question” and for the marking scheme to reward the intermediate steps, with no marks for a final answer only. Requiring intermediate steps to be set out could be a safeguard against illegitimate use of calculators. However, it would be necessary to ensure that the intermediate steps could not be done using a calculator, and we acknowledge that this proposal might raise regulatory problems.

Assessment

It is impossible to separate completely the content from other issues, in particular the assessment and structure are intimately interwoven with content. There is broad agreement within the mathematics community that the content of the current A level, as written, is roughly of the level and breadth required. However, it may be that the quality of assessment could be improved.

Common criticisms of the current examinations are:

- That they are perceived mainly to test speed and accuracy rather than actual mathematical ability.
- The examinations, at 1.5 hour duration, are too short to allow for in depth and searching questions.
- The examinations have become repetitive and predictable.
- Many questions are too highly scaffolded.

A common defence is that examinations are constrained by the need to provide a fair assessment across the whole range of attainment from grade E to A*.

The panel would like to see examinations which include some more searching questions. In the content we have increased the emphasis upon problem solving and upon deeper understanding through demonstrating interpretation. However, it would be necessary to see sample assessment materials and examinations in order to make more definitive proposals.

The panel's discussions have extended to wider issues including the impact of linear A levels, dangers to uptake, assessment, monitoring and implementation. Clearly these are outside the direct remit of this panel which was to consider solely the mathematical content of the A and AS levels. It is impossible completely to separate out the content from these wider issues and we have conveyed our views on the wider issues in a separate letter to the Department for Education.

List of recommendations

1. 100 per cent of the content for the single A level in mathematics should be fully prescribed centrally (page 6).

2. Students studying for A level mathematics should learn a variety of mathematical concepts, methods and techniques (page 6).

3. There should be a change in emphasis within the single A level in mathematics towards problem solving, interpretation and testing understanding (page 6).

4. Assessment should have less structured questions and should test understanding and help to develop strategies for solving problems either in a purely mathematical or in an applications context (page 6).

5. For further mathematics the panel considers that a core of 50% should be prescribed (page 7).

6. For the AS in further mathematics a minimum of 30% of the content must be drawn from the prescribed material of the A level in further mathematics and must include matrices and complex numbers (page 7).

7. The content currently contained in modules entitled “Decision Mathematics” should be completely removed from A level mathematics (page 8).

8. The use of real, large data sets should permeate the teaching, learning and assessment of statistics in AS and A level mathematics and further mathematics (page 8).

9. Assessment items in statistics should place more emphasis upon understanding, interpretation of data and making inferences from data than is found in the current content (page 8).

10. An emphasis upon understanding, interpretation and problem solving should pervade the teaching and assessment of mechanics (page 9).

11. During their study of mechanics students should build up understanding of the mathematical modelling process, including recognising the assumptions made and how these might be modified to improve the model used (page 8).

12. Examining organisations use common terminology between mathematics and physics where possible (page 9).

13. Technology should be used in both teaching and assessment (page 10).

14. Examinations should include some more searching questions (page 11)

15. Assessment tasks should reflect the recommended increased emphasis upon problem solving and upon deeper understanding through demonstrating interpretation (page 11).

ANNEX 1

List of panel members

Professor Richard Craster (Chair) Imperial College London
Dr Clare Dunning, University of Kent
Professor Peter Giblin, University of Liverpool & IMA
Professor Paul Glaister, University of Reading
Mr Mark Heslop, Altrincham Grammar School for Boys
Dr Steve Humble, University of Newcastle
Dr Sue Pope, University of Manchester
Mr Roger Porkess, independent
Professor Alice Rogers, King's College London & London Mathematical Society
Dr Louise Walker, University of Manchester
Dr Helen Wilson, University College London

ANNEX 2

List of organisations contributing to the panel's work

Advisory Committee on Mathematics Education

British Congress of Mathematics Education

Engineering Professors Council

Heads of Department of Mathematical Sciences

Institute of Mathematics and its Applications

Institute of Physics

London Mathematical Society

Operational Research Society

Royal Academy of Engineering

Royal Computing Society

Royal Economic Society

Royal Society of Chemistry

Royal Society

Royal Statistical Society

SCORE